This is a preliminary version of a review which will appear in *Research in the History of Economic Thought and Methodology*, Volume 24A (2006), edited by Warren Samuels, Jeff Biddle, and Ross Emmett.

Getting Hilbert Right: A Review of Leo Corry's *David Hilbert and the Axiomatization of Physics (1898 – 1918): From Grundlagen der Geometrie to Grundlagen der Physik*. Kluwer: Dordrecht and Boston, 2004.

By E. Roy Weintraub
Duke University

From the opening salvo of Benjamin Ward's (1972) *What's Wrong with Economics* to Henry Woo's (1986) *What's Wrong with Formalization in Economics* to Mark Blaug's (1999) "The formalist revolution or what happened to orthodox economics after World War II?", the specter of mathematician David Hilbert has haunted economists's discussions of formalization and axiomatization.

Briefly, if one looks upon formalized economics, or formalism, with a loathing built on fear (or a fear based on loathing), one demonizes Hilbert since the philosophical notion of formalism, in the history of metamathematics, is usually associated with Hilbert. That is, in the history of philosophy of mathematics, there appears to be a distinction between formalists and empiricists, on the nature of mathematical objects. The ontological reflections in that arcane literature has Hilbert holding the position that mathematics is simply a formal system, and its symbols are simply marks on paper. It is an easy step then to look for traces of David Hilbert in the development of mathematical economics in the 20th century. Seek, and ye shall find, and critics of mainstream economics have found Hilbertian connections in Vienna with Menger's seminar. As a result Hilbert gets caught up in the origin stories of general equilibrium theory which lead all the way to von Neumann and the development of game theory. From Vienna and general equilibrium theory it is a short step, though a false step, to have Hilbert as the spiritual advisor to the Cowles Commission and thence to Arrow-Debreu. From there of course one can launch tirades about the formalist revolution in economics and have Hilbert bearing some of the blame for a misguided economics.

This view of Hilbert is near total nonsense. Ewald (1996, p. 1106) said it best: "As for the term `formalist', it is so misleading that it should be abandoned altogether as a label for Hilbert's philosophy of mathematics." Moreover, the notion of Hilbert as formalist is often connected to his 1917 talk, then paper (Hilbert 1918), on axiomatization, which leads the "It's Hilbert's fault" crowd to think of him as the father of Arrow's (1951) *Social Choice and Individual Values* and Debreu's (1959) *Theory of Value*. This too is nonsense. Serious historians of mathematics have long known that these characterizations of Hilbert, developed out of bad history of the philosophy of mathematics, misread Hilbert and his role in both physics and mathematics. Historians of economics have not seemed to understand these matters.

The origin of the more comprehensive, and historically accurate, account of Hilbert's work on axiomatization has its roots in work by several historians of mathematics, particularly David Rowe in a series of articles written in the past decade and a half (e.g. Rowe 1997), and more directly with articles by Leo Corry in the 1990s, and now expanded and worked out in a great deal more detail in his new book, *David Hilbert and the Axiomatization of Physics (1898 – 1918)*. Corry, an important historian of 20th century mathematics, begins his discussion of Hilbert and axiomatization with a period of time in the latter part of the 19th century when Hilbert was working both on matters of the foundations of geometry as well as the development of research tools in physics. Far from Hilbert's being a mathematician primarily interested in axiomatization as foundationalist work in mathematics, Corry shows how Hilbert was, from the beginning of his career, fully involved with the role of mathematics in facilitating research in physics.

A historian of economics cannot do real justice to the fine details of Corry's narrative, but certainly in it one can admire Corry's intensive archival research, his sympathetic comprehension of the large canvas on which Hilbert worked, and his demonstration of the benefits to the historian of understanding in detail all not only the "Hilbert-ian texts", but the situation of all of these ideas in the particular times and places of their genesis. It is a model history of science. Nevertheless, there is a great deal here that touches, if only peripherally, on a set of issues that have concerned economists, specifically historians of mathematical economics, for a number of years.

As is well known, Hilbert's 23 problems from his 1900 World Congress lecture contained among them the 6th problem, concerning the appropriate axiomatization for physics. It is not as if people failed to understand that Hilbert was interested in axiomatization of physical theories, but rather, as Corry argues, that Hilbert's concern with mathematics as an engine of discovery in science, particularly in physics, was pushed aside by historians of metamathematics more concerned with Hilbert as an ontological formalist. It is not only economists who reconstruct historical figures to make them allies in current controversies.

Using Hilbert's lecture notes from that 1900-1910 period, preserved at Göttingen, Corry's Hilbert emerges as an individual concerned with axiomatization *because* by axiomatizing the physical theories, one could locate exactly the strong and weak points of the physical argument. Put another way, one's axioms are not chosen haphazardly, but an axiomatization must be based on real knowledge of the underlying physical models. That said, the view of axiomatization fixed in some economists' minds, namely that axioms are "divorced from reality," is absolutely wrong headed. This should have been clear from a close reading of Hilbert's talk and paper (1918) "Axiomatisches Denken", but apparently it was not.

Corry suggests that

> "The most noteworthy feature of this talk is the demarcation introduced here for the first time between two kinds of systems of axioms that I will call, for want of a better name, pragmatic and foundational. Pragmatic axioms are those that underlie established fields of knowledge that have already become elaborate theories or

> network[s] of concepts. The axioms allowed deriving the main theorems of these theories . . . the role of the axioms is, then, to provide 'an initial standpoint,' and 'the progressive development of the individual field of knowledge then lies solely in the further logical construction of the already mentioned framework of concepts.' But then Hilbert stressed a new perspective that he had never mentioned so far in this context, at least not in this way: the solution provided by such axioms and grounding their respective fields of knowledge, he said, turns out to be only temporary, and they stand in need of being themselves further grounded." (396 – 397)

What Corry has achieved in this book is a repositioning of David Hilbert. No longer is he the simple figure of the metamathematician's history, nor now even just a towering figure in the history of mathematics. Corry's Hilbert emerges as a significantly more complex and "situated" figure. From Chapter 1 which locates Hilbert's concerns in algebra and geometry, to a set of connections with physicists like Volkmann and Boltzmann, Corry moves to the connection of mathematics and physics by considering Hilbert and Felix Klein. The second chapter takes up the emergence of Hilbert's views on axiomatization in both geometry and physics in his early lectures, and moves then to his 1893-1894 *Grundlagen der Geometrie*, which may be taken as a signal marker in the development of axiomatization in mathematics. Corry goes on to situate axiomatization both in mathematics and in physics in Hilbert's 1900 problem list, and moves in Chapter 3 to showing, based on his recovery of Hilbert's 1905 lectures, the roles axioms played for Hilbert in physical theories, specifically in mechanics, thermodynamics, the probability calculus, the kinetic theories of gases, insurance mathematics (!), electrodynamics, and psychophysics. That Hilbert continued to develop these ideas about axiomatization in physics is the concern of Corry in his Chapter 4, looking at his 1907 to 1909 work on relativity. In his Chapter 5, Corry examines the 1910-1914 discussions of the move from mechanical to electro magnetic reductionism. Both Chapters 6 and 7 concern the foundation of physics, and Chapter 8 presents Hilbert's forays into the general theory of relativity. All of these concerns of course are associated with Hilbert's program for axiomatization of physical theories

The David Hilbert that emerges in this detailed study was concerned in approximately the first two decades of the 20th century to establish ways in which research in physics could be moved forward. Recall that the issues of black body radiation, and relativity, were roiling the physics community. What was solid knowledge and what was discredited physics were open questions. For Hilbert, the role of axiomatization was to lay bare the structure of the known physical theory and the phenomena so that implications could be brought forward. Such a view makes Hilbert a sympathizer with those who wish to see economic models as "reality based." However, for a model to be reality based does not mean that the model is nonmathematical, or necessarily presented in a nonaxiomatized version.

Did Hilbert's perspective change the way research in physics was done? Corry's conclusion is circumspect here:

> "Whether or not physicists should have looked more closely at Hilbert's ideas than they actually did, and whether or not Hilbert's

> program for the axiomatization of physics had any influence on subsequent developments in this discipline, it is important to stress in any case that a full picture of Hilbert's *own* conception of mathematics cannot be complete without taking into account his views on physical issues and on the relationship between mathematics and physics. More specifically, a proper understanding of Hilbert's conception of the role of the axioms in physical theories helps us to understand his conception of the role of axioms in mathematical theories at large. The picture that arises from such an understanding is obviously very far away from the once widespread image of Hilbert as the champion of the formalistic conception of the nature of mathematics." (443)

Corry's book, while probably not much to the taste (or likely comprehension) of many economists, nevertheless is a signal achievement as it intertwines the history of mathematics and the history of mathematical physics. Now that Corry has exhibited the ideas about formalization, formal systems, and axiomatization in their natural habitat, economists can no longer ignore the implications of this connection between mathematicians' ideas, specifically their changing ideas about axiomatization, and the concomitant changing ideas of economists about axiomatization and rigor.

References

.

Arrow, K. J. (1951). Social Choice and Individual Values. New York, John Wiley and Sons.

Blaug, M. (1999). The formalist revolution or what happened to orthodox economics after World War II? From Classical Economics to the Theory of the Firm. R. E. Backhouse and J. Creedy. Cheltenham, UK, Edward Elgar.

Debreu, G. (1959). The Theory of Value. New York, John Wiley.

Ewald, W., Ed. (1996). From Kant to Hilbert: A Source Book in the Foundations of Mathematics: Volume II. Oxford and New York, Oxford University Press.

Hilbert, D. (1918). "Axiomatisches Denken." Mathematische Annalen **78**: 405-415.

Rowe, D. E. (1997). "Perspective on Hilbert." Perspectives on Science **5**(4): 533-570.

Ward, B. (1972). What's Wrong with Economics. New York, Basic Books.

Woo, H. K. H. (1986). What's Wrong With Formalization in Economics? An Epistemological Critique. Newark, CA, Victoria Press.